$$\therefore \frac{56ax}{27}+\frac{16a}{27}(x-9)=8(a+18an)=40a, \text{ whence } x=22, \text{ number required.}$$

Also solved by *J. R. HITT.*
Solutions of problem 152 were received from *J. K. ELLWOOD* and *J. R. HITT.*

ALGEBRA.

NOTE ON SOLUTION I. OF PROBLEM 131.

In connection with this solution it is well to note that Dr. Zerr's results hold only for the case where a, b, c, and d are all positive, and a^2d+b^2c-cd is positive, restrictions which he neglects to mention. For instance, the equation $2x-1+\sqrt{(5x^2+1)}=0$ possesses two roots, 0 and -4, yet $c>a^2$ or $5>2^2$.

The criteria of the second solution cover all real values of a, b, c, and d, positive or negative, and a^2d+b^2c-cd positive.

H. S. VANDIVER, Bala, Pa.

133. Proposed by HARRY S. VANDIVER, Bala, Pa.

A theory of Fermat. The sum of two integral fourth powers cannot be an integral square. [Cf. *Chrystal's Algebra*, Vol. II, page 535.]

I. Solution by L. C. WALKER, A. M., Professor of Mathematics, Petaluma High School, Petaluma Cal., and CHAS. C. CROSS, Whaleyville, Va.

Assume m and n to be the numbers, then by the condition of the problem, we have

$$(m^2)^2+(n^2)^2=c^2, \text{ suppose}\ldots\ldots(1).$$

In (1) either m^2 or n^2 must be even, and the other odd, because the sum of two odd squares can not be a square.

Assume m^2 even and n^2 odd.

Let $m^2=2pq$ and $n^2=p^2-q^2$; or $n^2+q^2=p^2\ldots\ldots(2)$.

In (2) n and p are odd and q even. Now let $q=2\alpha\beta$ and $p=\alpha^2+\beta^2$. Then we find $m^2=4\alpha\beta(\alpha^2+\beta^2)\ldots\ldots(3)$.

Since α and β are prime to each other, in order that $\alpha\beta$ may be a square, each must be a square.

Let $\alpha=m_1^2$ and $\beta=n_1^2$. Substituting in (3), we get

$$m^2=4m_1^2n_1^2(m_1^4+n_1^4)\ldots\ldots(4).$$

In order that the right member of (4) may be a square, we must have $m_1^4+n_1^4=c_1^2$, say; which is of the same form as (1). But c_1^2 is less than c^2.

Proceeding in exactly the same way, we can reduce c^2 indefinitely by some integer. By our hypothesis c^2 can not be zero, nor less.

Hence, results *Fermat's Theorem.*

Also solved by *G. B. M. ZERR.*

II. Remarks by the PROPOSER.

Euler's proof of this theorem which appears in his treatise on Algebra, I believe to be the simplest demonstration that can possibly be obtained. For the benefit of those who are unacquainted with it, I reproduce the substance of the argument below.

EULER'S PROOF. Suppose that three integers a, b, and c are found such that $a^4+b^4=c^2$. If a and b are not prime to each other let $a=a'y$, and $b=a'x$, then $(a'y)^4+(a'x)^4=c^2$. Whence, by division, $x^4+y^4=\square=z^2$, suppose.

Hence it is sufficient to consider the case

$$x^4+y^4=z^2\ldots\ldots(1),$$

where x, y and z are prime to each other. If this relation be satisfied, then it follows that there are integers p and q such that

$$\left.\begin{array}{l}x^2=2pq\\ y^2=p^2-q^2\end{array}\right\}\text{ or }\left\{\begin{array}{l}x^2=p^2-q^2\\ y^2=2pq\end{array}\right.$$

Since x and y are symmetrical in (1) it is sufficient to put

$$x^2=2pq\ldots\ldots(2),\ y^2=p^2-q^2\ldots\ldots(3).$$

Since x is prime to y and x is evidently even, it follows that y is odd, and y^2 must then be of the form $4n+1$. In (3) we have the following hypotheses for p^2 and q^2:

$$\begin{array}{l}p^2=4m\text{ or }4m+1\\ q^2=4k\text{ or }4k+1\end{array}$$

If $p^2=4m$ and $q^2=4k$ then $y^2=4(m-k)$, which is impossible, since y is odd. Put $p^2=4m$ and $q^2=4k+1$; then $y^2=4(m-k)-1$, which is also absurd. $\therefore\ p^2=4m+1$ and $q^2=4k$, whence p is odd and q is even.

Therefore integers r and s can be found so that $p=r^2+s^2$, $q=2rs$, where r and s are prime to each other.

Substituting in (2) we have

$$x^2=4rs(r^2+s^2)\text{ or }x_1^2=rs(r^2+s^2).$$

Since r, s and r^2+s^2 are all prime to each other we will have $r=r_1^2$, $s=s_1^2$. Then also, $r^2+s^2=r_1^4+s_1^4=\square=u_1^2$, say. Now in $r_1^4+s_1^4=u_1^2$, $r_1<r<x$.

That is, supposing (1) to hold, we have shown that it is possible to find a relation $r_1^4+s_1^4=u_1^2$ so that $r_1<x$.

In the same manner it may be shown that we can obtain $r_2^4+s_2^4=u_2^2$, where $r_2<r_1$, and evidently, $r_n^4+s_n^4=u_n^2$ where $r_n<r_{n-1}<r_{n-2}\ldots\ldots<r_1<x$.

So that by taking n sufficiently large it will be found

$$1+s_n^4=u_n^2\ldots\ldots(4),$$

whence $u_n^2-s_n^4=1$, and $u_n=1$ and $s_n=0$ are the only solutions. Now s_n cannot be 0 for if such were the case we would have

$$s_n=s_{n-1}=s_{n-2}\ldots\ldots=s_1=x=0,$$

which is inconsistent with the definition of x. Hence the impossibility of (4) and therefore of (1) is completely demonstrated.

134. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Solve neatly and briefly the equations

$$x^3+x^2y+y^3=53\ldots.(1),\ y^3+y^3z+z^3=13\ldots.(2),\ \text{and}\ z^3+z^2x+x^3=31\ldots.(3).$$

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

Probably the only brief solution is by inspection, as follows:

$$x^3+x^2y+y^3=53=27+18+8=3^3+2.3^2+2^3.$$

$$y^3+y^2z+z^3=13=8+4+1=2^3+1.2^2+1^3.$$

$$z^3+z^2x+x^3=31=1+3+27=1^3+3.1^2+3^3.$$

$\therefore\ x=3,\ y=2,\ z=1.$

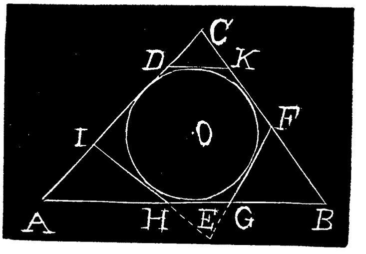
135. Proposed by CHARLES C. CROSS, Whaleyville, Va.

Tangents parallel to the three sides are drawn to the in-circle. If p, q, r, be the lengths of the parts of the tangents within the triangle, prove that

$$\frac{p}{a}+\frac{q}{b}+\frac{r}{c}=1.$$

I. Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.

Let ABC be any triangle with the in-circle O. Put $AB=c$, $AC=b$, $BC=a$.



Draw the respective tangents, $DK=r$, parallel to AB; $FG=q$, parallel to AC; and $HI=p$, parallel to BC.

Let $AH=x$, and $BG=y$.

The construction of lines and similarity of triangles give the following:

$HG=DE=r$; $y:c=q:b$, or $y=cq/b$; and $x:c=p:a$, or $x=cp/a$. But $x+y+r=c$.

$$\therefore\ \frac{cp}{a}+\frac{cq}{b}+r=c;\ \text{or}\ \frac{p}{a}+\frac{q}{b}+\frac{r}{c}=1.$$

Solved in a similar manner by *LON C. WALKER* and *J. SCHEFFER*.